Multiscale partitioning of available potential energy in energetically consistent models

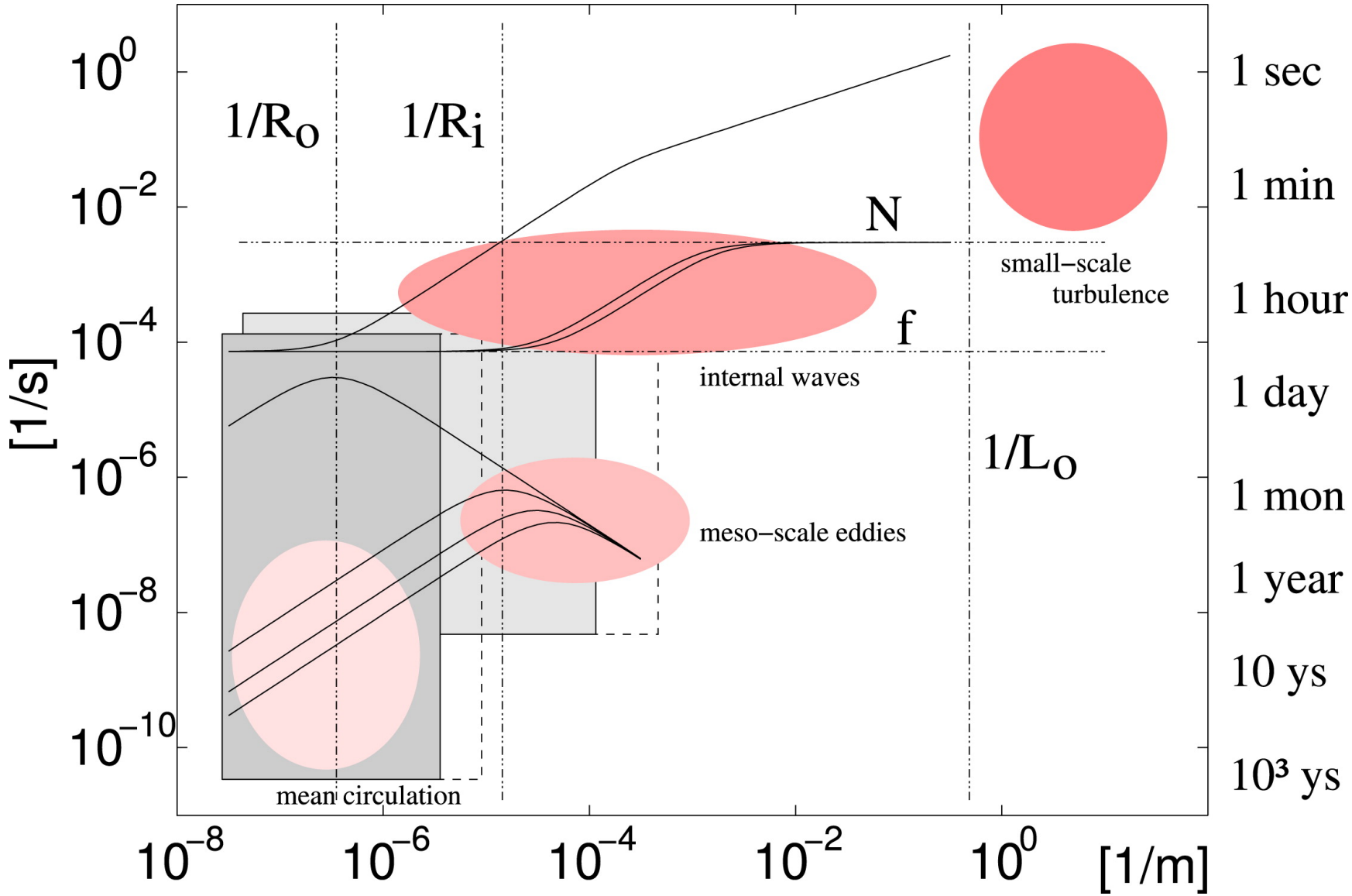
Rémi Tailleux

Energy Transfers Meeting, Hamburg, 4-7 April 2018

Energetically Consistent Ocean Models

- Models with explicit description of energy transfers between resolved and unresolved energy reservoirs down to the dissipation scales
- Main aim: Ensure that mixing coefficient have correct dependence on energy sources of stirring to ensure correct sensitivity to climate change
- Questions & Challenges:
  - How to meaningfully partition potential energy, which is linear rather than quadratic in perturbations?
  - Are the energy pathways associated with turbulent diapycnal mixing correctly understood?

Eden, Czechel and Holbers (JPO, 2014)



KINETIC ENERGY IS EASY, POTENTIAL ENERGY IS HARD

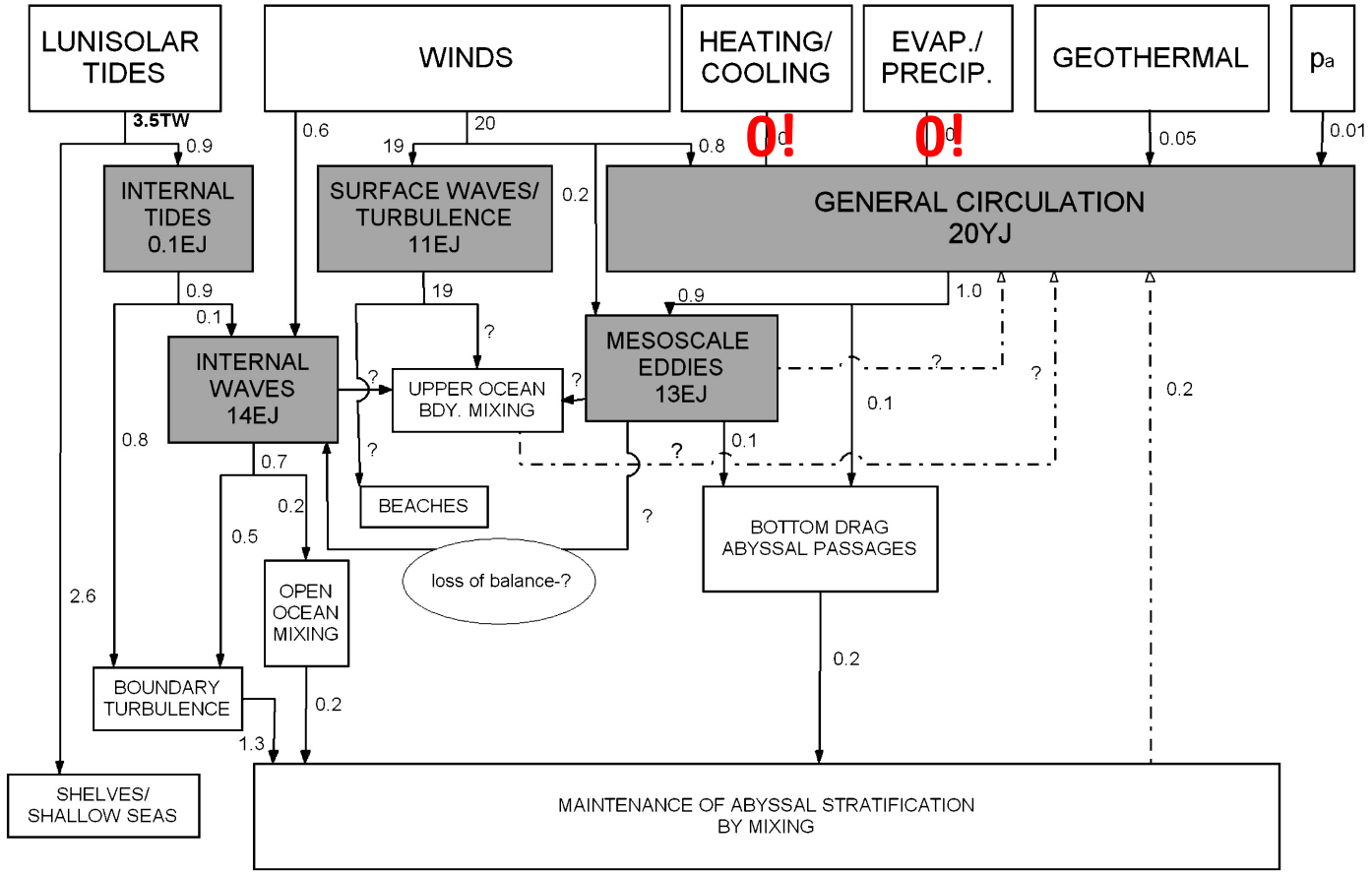
We know how to partition kinetic energy into subcomponents, or to define kinetic energy spectra:

$$\frac{\mathbf{v}^2}{2} = \frac{\mathbf{v}_m^2}{2} + \frac{\mathbf{v}_e^2}{2} + \mathbf{v}_m \cdot \mathbf{v}_e$$

We know that potential energy can be partitioned into an available part and a background part, but we have struggled to produce meaningful partitioning into subcomponents or define spectra save for approximate expressions of APE

Dangers of using only PE rather than APE, e.g., Wunsch & Ferrari (2004): you get something stupid!

Buoyancy forcing does not supply any power to the ocean!



OUTLINE

- Main results from Local theory of available potential theory for a multicomponent compressible stratified fluid from Tailleux (JFM, Rapids 2018)
- Recent results about backbround potential energy as the best definition of heat
- The different flavours of heat, and the nature of Mixing and Available Potential Energy dissipation

Making sense of potential energy, available potential energy and background potential energy

Potential energy is the energy required to construct the stratification from scratch

(Scratch = Thermodynamic Equilibrium)

Effective potential energy

PV work due to environment

$$B = \phi(z) + e(S, \eta, p) + \frac{p_0(z)}{\rho(S, \eta, p)}$$

Geopotential Height = g z

Internal Energy

In absence of viscous and diabatic effects:

$$\rho \frac{D}{Dt}\left(\frac{\mathbf{v}^2}{2} + B\right) + \nabla \cdot [(p - p_0)\,\mathbf{v}] = 0$$

Main result of Tailleux (JFM Rapids, 2018)

$$\Pi = B - B_R \geq 0$$

Effective potential energy minus reference value is naturally positive definite!

Small Amplitude Approximation

$$\Pi \approx \frac{\left(p - p_0(z)\right)^2}{2\,\rho^2 c_s^2} + \frac{N_R^2 (z - z_R)^2}{2}$$

Available Elastic Energy (AEE) Density

Available Potential Energy (APE) Density

THEOREM ('Mean/Eddy' partition)

In the same way that kinetic energy can be decomposed exactly as

$$\frac{\mathbf{v}^2}{2} = \frac{\mathbf{v}_m^2}{2} + \frac{\mathbf{v}_e^2}{2} + \mathbf{v}_m \cdot \mathbf{v}_e$$

it is similarly possible to decompose APE density exactly as

$$APE\ density = \Pi_m + \Pi_e + \delta\Pi$$

where the first two terms are positive definite, for arbitrarily defined mean and eddy reference density/pressure profiles

$$\Pi_e = \Phi(z) - \Phi(z_{rm}) + e(\eta, S, p) - e(\eta, S, p_{rm}) + \frac{p_m(x, y, z)}{\rho} - \frac{p_{rm}}{\rho_{rm}}, \tag{4.1}$$

$$\Pi_m = \Phi(z_{rm}) - \Phi(z_r) + e(\eta, S, p_{rm}) - e(\eta, S, p_r) + \frac{p_0(z_{rm})}{\rho_{rm}} - \frac{p_r}{\rho_r}, \tag{4.2}$$

$$\delta\Pi = \frac{p_0(z) - p_m(x, y, z)}{\rho} + \frac{p_m(x, y, z_{rm}) - p_0(z_{rm})}{\rho_{rm}}. \tag{4.3}$$

$p_0(z), \rho_0(z) \quad = \quad Resting\ Reference\ State$

$p_m(x, y, z), \rho_m(x, y, z) = \quad Non - Resting\ Reference\ State$

Interactions between mean, eddy and interaction energies

$$\rho \frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{\boldsymbol{v}^2}{2}+\Pi_e\right)+\nabla\cdot\{[p-p_m(x,y,z)]\boldsymbol{v}\}=-\frac{\rho\boldsymbol{u}\cdot\nabla_h p_{rm}}{\rho_{rm}}, \tag{4.4}$$

$$\rho \frac{\mathrm{D}\Pi_m}{\mathrm{D}t}+\nabla\cdot\left\{\frac{\rho[p_{rm}-p_0(z_{rm})]\boldsymbol{v}}{\rho_{rm}}\right\}=\frac{\rho\boldsymbol{u}\cdot\nabla_h p_{rm}}{\rho_{rm}}, \tag{4.5}$$

$$\rho \frac{\mathrm{D}(\delta\Pi)}{\mathrm{D}t}=\nabla\cdot\left\{\left[p_0(z)-p_m(x,y,z)+\frac{\rho(p_{rm}-p_0(z_{rm}))}{\rho_{rm}}\right]\boldsymbol{v}\right\}, \tag{4.6}$$

- Energy transfer between mean and eddy fields vanish for purely geostrophic reference state
- Volume integral of interaction potential energy vanishes

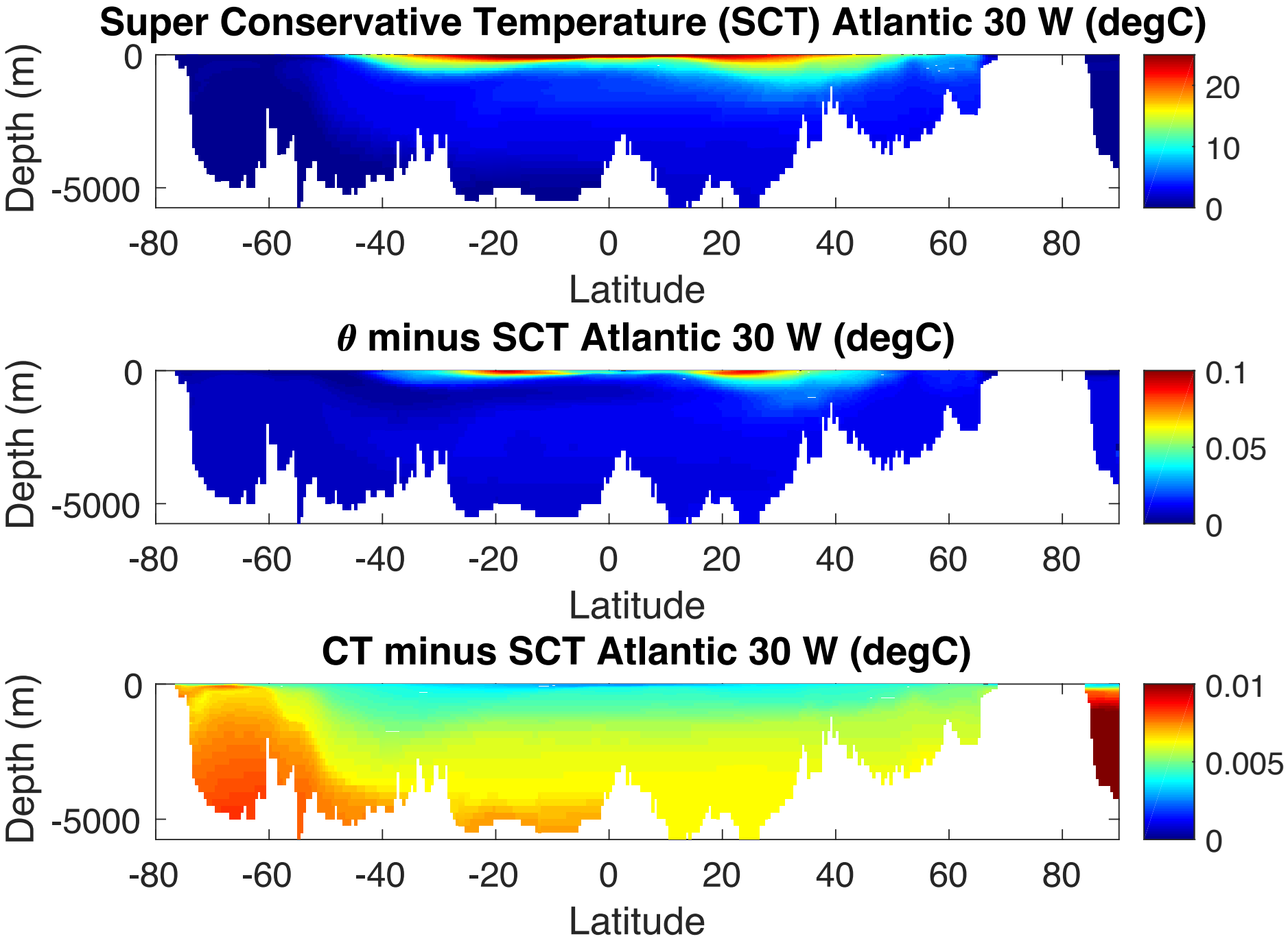
Background Potential Energy B_R

Can be written as:

$$B_R = h\big(S, \eta, p_0(z_R)\big) + \phi(z_R)$$

Can be shown:

- To be more conservative than potential enthalpy/Conservative Temperature. In fact, it is the most conservative heat variable.
- Background potential energy is hence the natural definition of ‘heat’, and Available Potential energy the most natural definition of ‘work’.


Super Conservative Temperature (SCT) Atlantic 30 W (degC)
Depth (m)
0
-5000
-80
-60
-40
-20
0
20
40
60
80
Latitude
20
10
0
θ minus SCT Atlantic 30 W (degC)
Depth (m)
0
-5000
-80
-60
-40
-20
0
20
40
60
80
Latitude
0.1
0.05
0
CT minus SCT Atlantic 30 W (degC)
Depth (m)
0
-5000
-80
-60
-40
-20
0
20
40
60
80
Latitude
0.01
0.005
0

Background Potential Energy BR (cont'd)

$$B_R \approx \frac{\partial \mu}{\partial S}\frac{(S - S_*(p_R))^2}{2}$$

>0 Haline Exergy = Energy needed to construct salinity stratification

$$+ \frac{c_p}{T_*}\frac{(T_R - T_*)^2}{2}$$

>0 Thermal Exergy = Energy needed to construct thermal stratification

$$+ \int_{p_m}^{p_R} (v_*(p') - v_R(p'))dp'$$

<0 Loss in gravitational potential energy resulting from `un-mixing' stratification

$$+ T_*\eta + \mu_* S + constant$$

Dead Internal Energy = Internal energy of fluid at thermodynamic equilibrium

Background Potential Energy BR (cont'd)

$$B_R \approx \frac{\partial \mu}{\partial S} \frac{(S - S_*(p_R))^2}{2}$$

0 to 0.04 C (40 mC)

$$+ \frac{c_p}{T_*} \frac{(T_R - T_*)^2}{2}$$

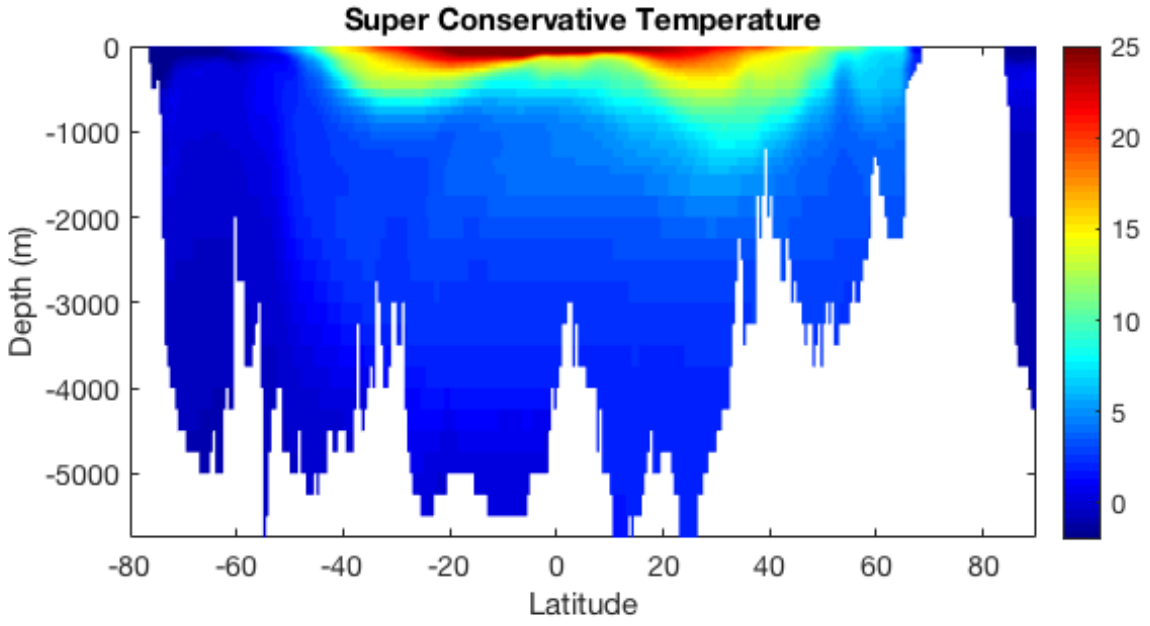
0 to 1C (0 to 0.04 C in the deep ocean)

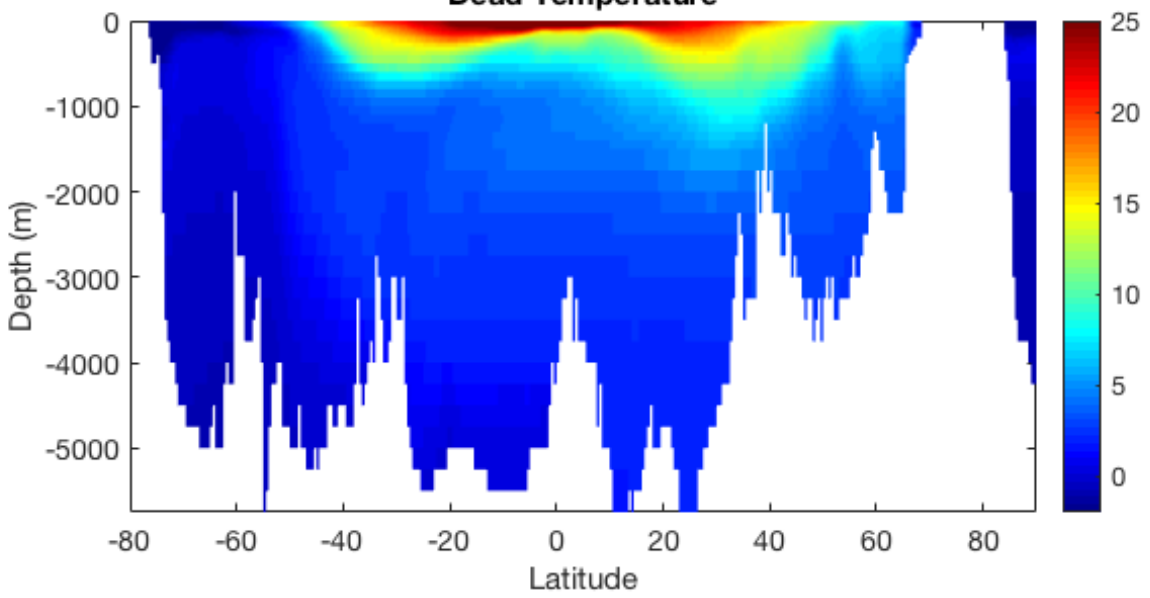
$$+ \int_{p_m}^{p_R} (v_*(p') - v_R(p'))dp'$$

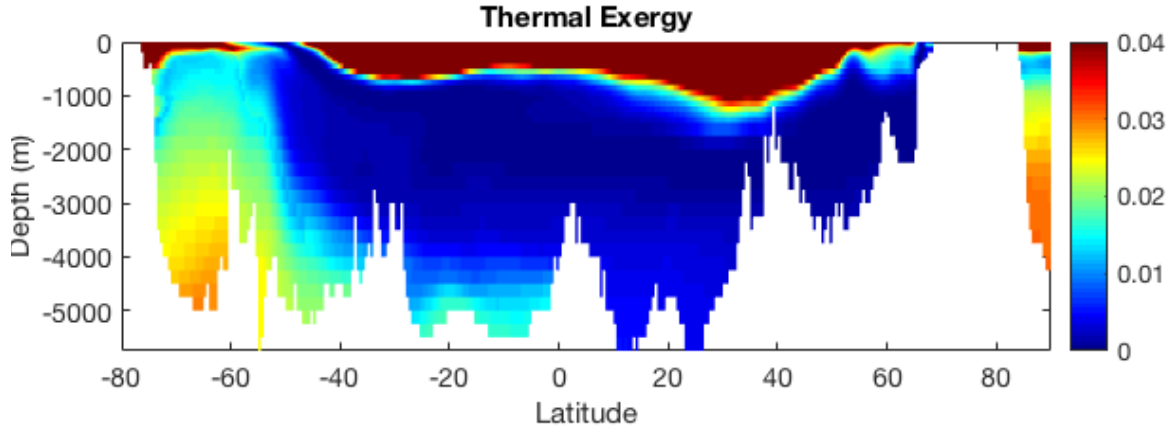
0 to 0.03 C (30 mC)

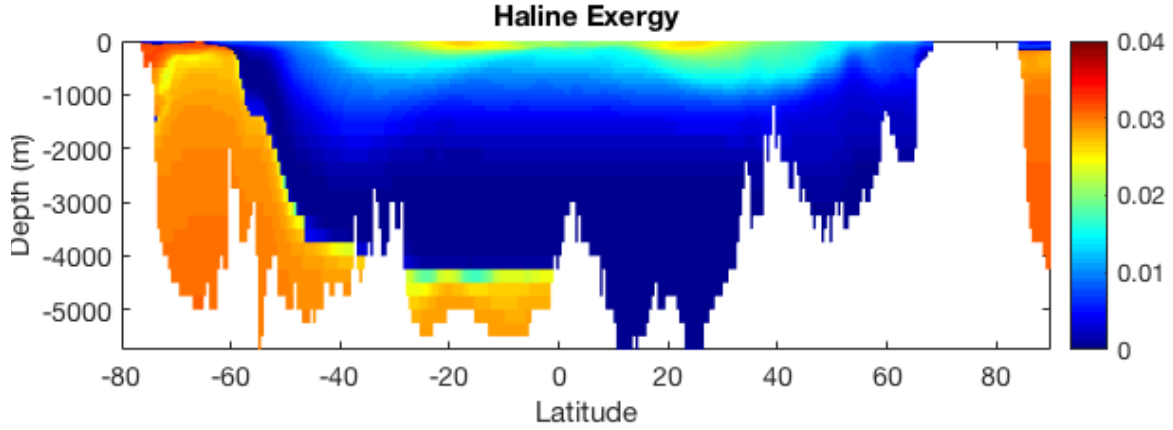
$$+ T_*\eta + \mu_* S + constant$$

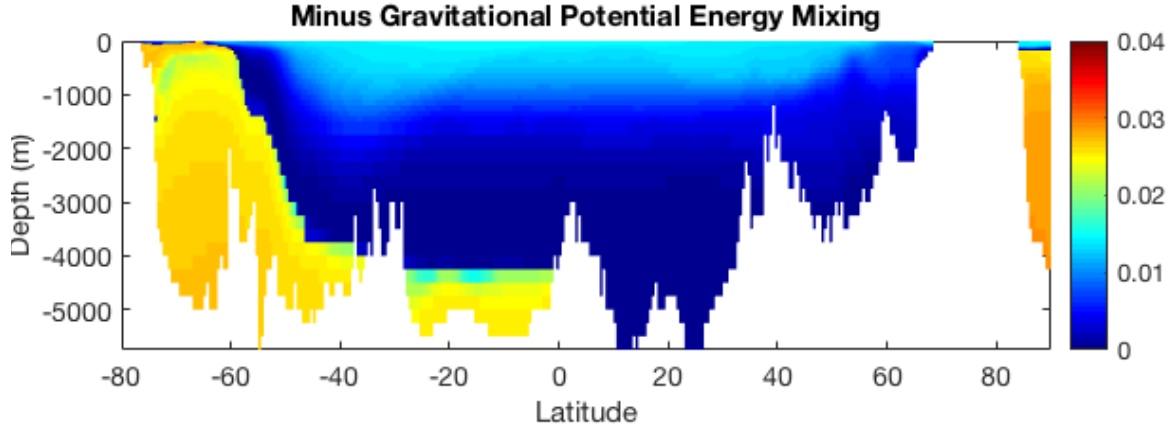
-2 C to 25 C


Super Conservative Temperature
Depth (m)
Latitude
0
-1000
-2000
-3000
-4000
-5000
-80
-60
-40
-20
0
20
40
60
80
25
20
15
10
5
0


Dead Temperature
Depth (m)
Latitude
0
-1000
-2000
-3000
-4000
-5000
-80
-60
-40
-20
0
20
40
60
80
25
20
15
10
5
0


Thermal Exergy
Depth (m)
Latitude
0
-1000
-2000
-3000
-4000
-5000
-80
-60
-40
-20
0
20
40
60
80
0.04
0.03
0.02
0.01
0


Haline Exergy
Depth (m)
Latitude
0
-1000
-2000
-3000
-4000
-5000
-80
-60
-40
-20
0
20
40
60
80
0.04
0.03
0.02
0.01
0


Minus Gravitational Potential Energy Mixing
Depth (m)
Latitude
0
-1000
-2000
-3000
-4000
-5000
-80
-60
-40
-20
0
20
40
60
80
0.04
0.03
0.02
0.01
0

Thermodynamic transformations required to construct the oceanic stratification

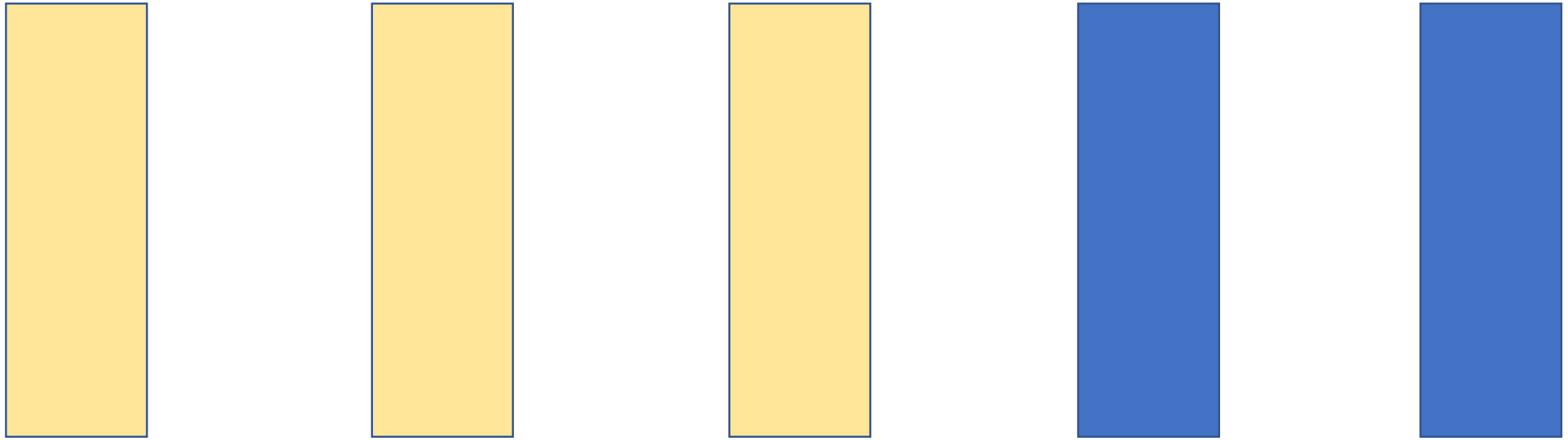
Scratch = Thermodynamic Equilibrium

Isothermal and isobaric redistribution of salt

Isohaline and isobaric heating/cooling

Adiabatic and isohaline displacement to reference pressure

Adiabatic/isohaline compression/ expansion to actual pressure



Dead Heat

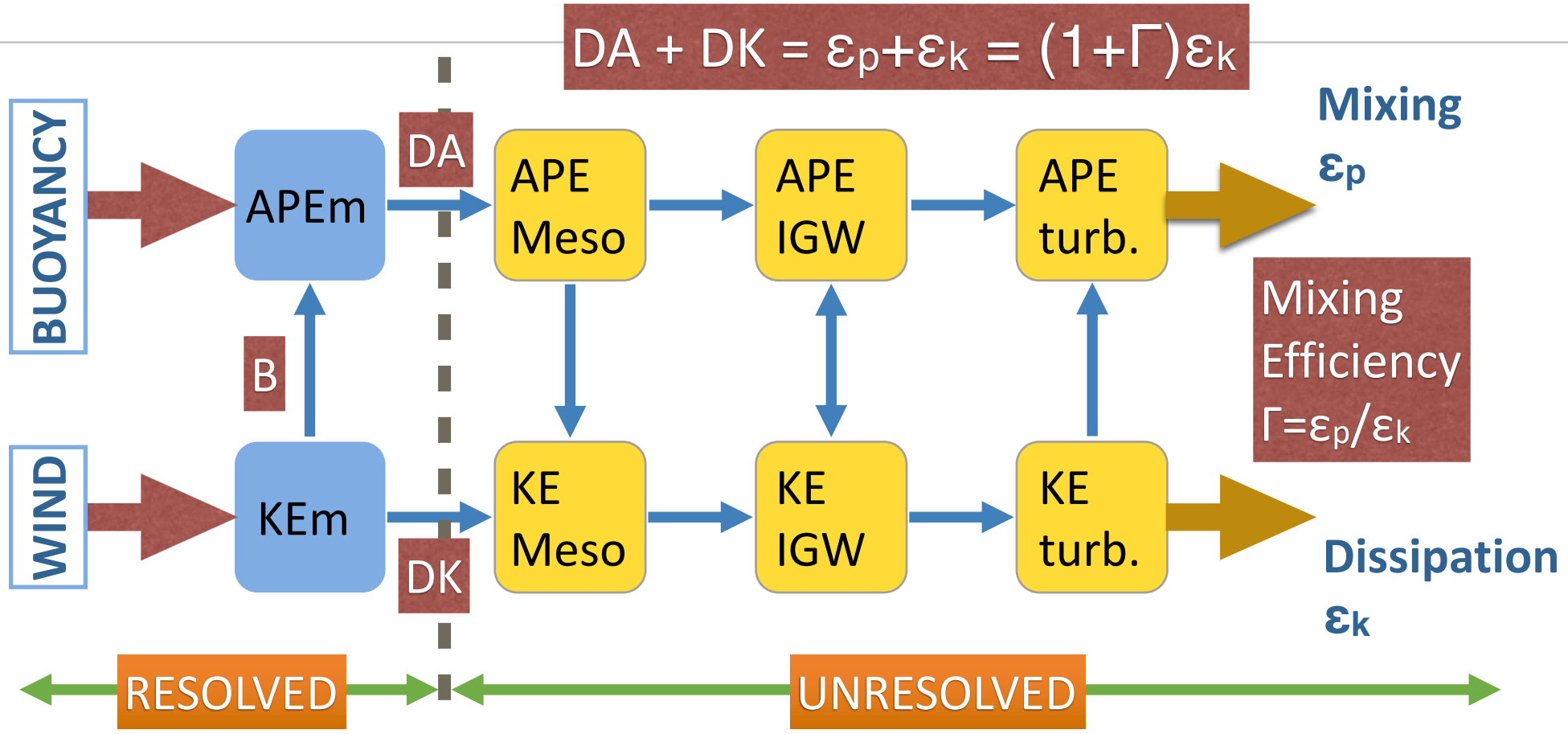
Salinity variance Exergy

Temperature Variance exergy

Available Potential Energy (APE)

Available Elastic Energy (AEE)

Resolved and Unresolved Energy pathways - COARSE RESOLUTION MODELS



Available Potential Energy Dissipation ε_p

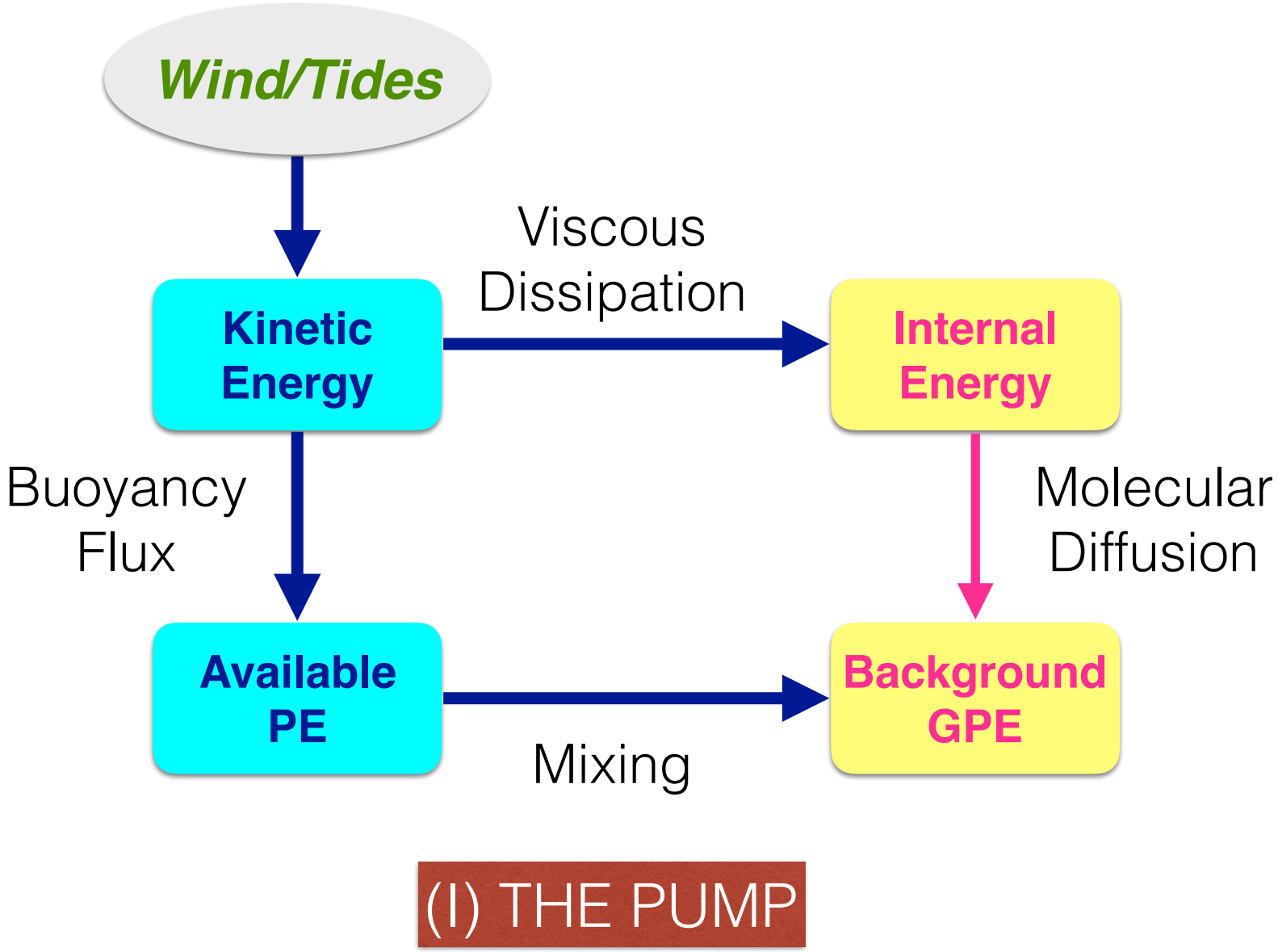
Reversible or Irreversible?

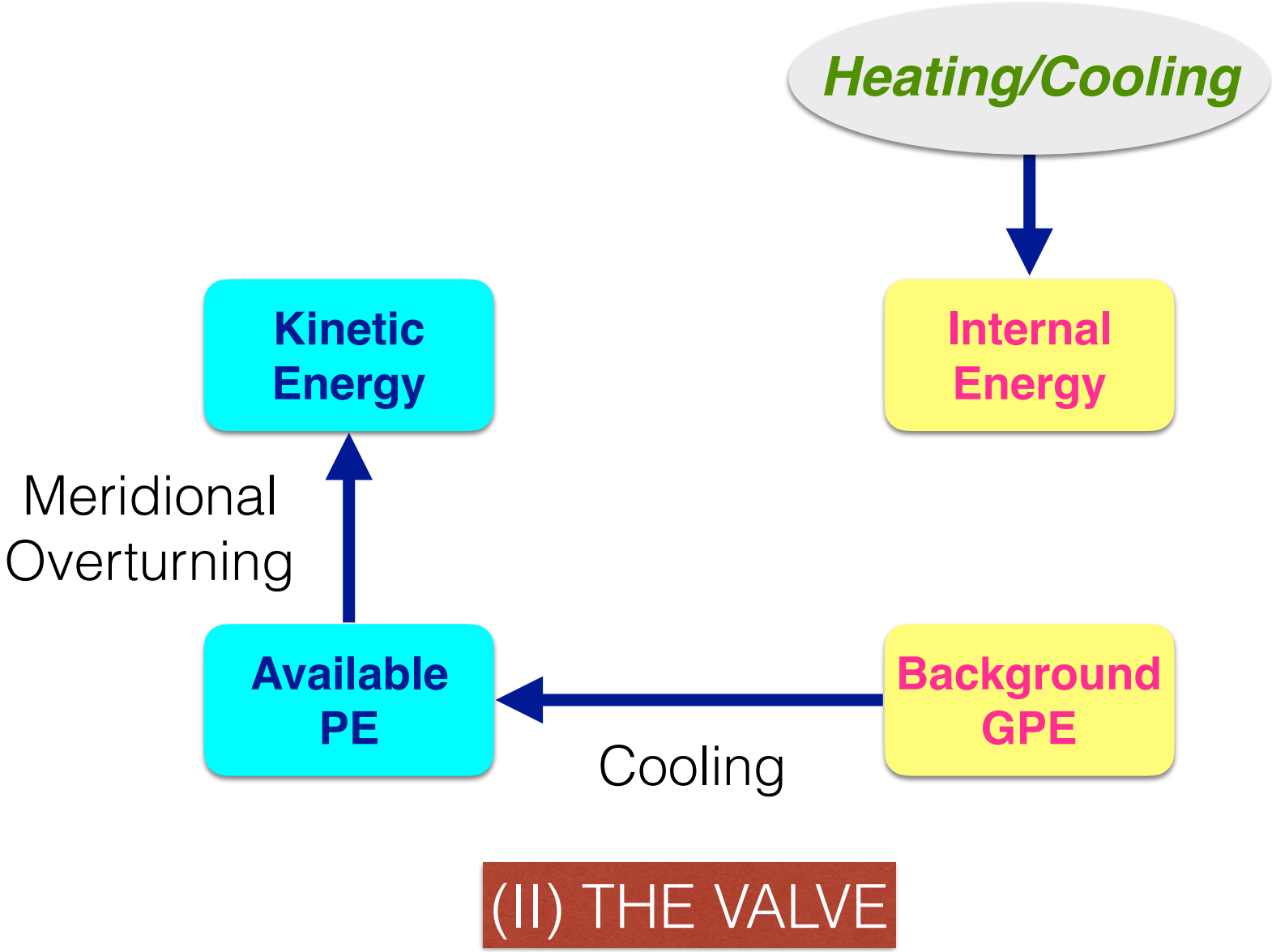
$$\varepsilon_p = K_v N^2 = \Gamma \varepsilon_K$$

Turbulent diapycnal diffusivity

Mixing Efficiency

- Systematically referred to as an `irreversible' energy conversion
- Yet, energy pathways for APE dissipation by Hughes et al. (2009) and Winters et al. (1995) describe a reversible energy conversion
- **Mixing converts APE into background gravitational potential energy, surface cooling converts it back to APE: The Pump/Valve mechanism**


Wind/Tides
Kinetic Energy
Viscous Dissipation
Internal Energy
Buoyancy Flux
Molecular Diffusion
Available PE
Background GPE
Mixing
(I) THE PUMP


Heating/Cooling
Internal Energy
Kinetic Energy
Meridional Overturning
Available PE
Background GPE
Cooling

(II) THE VALVE

some algebra yields:

$$\rho \dot{\Pi} = -\nabla \cdot (\rho \mathbf{J}_\Pi) - \rho \varepsilon_p, \tag{2.29}$$

where as usual, the reversible component of $\rho\dot{\Pi}$ is the divergence term, whereas its irreversible component is $-\rho\varepsilon_p$. Here, $\mathbf{J}_\Pi$ represents the diffusive flux of Π,

$$\mathbf{J}_\Pi = (T - T_r)\mathbf{J}_\eta + (\mu - \mu_r)\mathbf{J}_s, \tag{2.30}$$

while ε_p is our sought-for expression for APE dissipation, given by

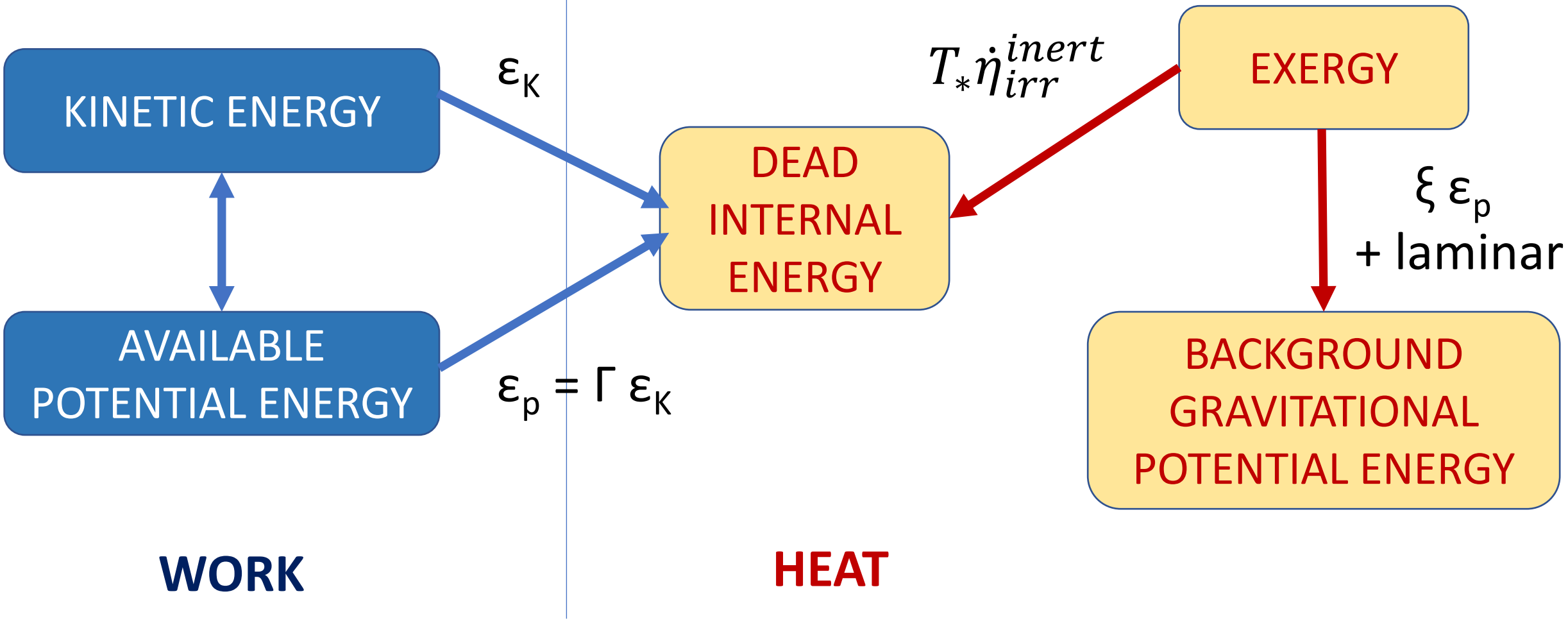
$$\varepsilon_p = -T_r \left[\mathbf{J}_\eta \cdot \left(\frac{\nabla T}{T} - \frac{\nabla T_r}{T_r} \right) + \mathbf{J}_s \cdot \left(\frac{\nabla \mu}{T} - \frac{\nabla \mu_r}{T_r} \right) + \frac{\Upsilon \varepsilon_k}{T_r} \right]. \tag{2.31}$$

Eq. (2.31) generalises (1.1) previously obtained for a simple fluid by Tailleux (2013*b*); it also allows for the irreversible entropy production term (2.5) to be rewritten in the form:

$$\rho \dot{\eta}_{irr} = \underbrace{-\rho \left(\mathbf{J}_\eta \cdot \frac{\nabla T_r}{T_r} + \mathbf{J}_s \cdot \frac{\nabla \mu_r}{T_r} \right)}_{\rho \dot{\eta}_{irr}^{inert}} + \underbrace{\frac{\rho(\varepsilon_p + \varepsilon_k)}{T_r}}_{\rho \dot{\eta}_{irr}^{active}}. \tag{2.32}$$

Actual Energy Pathways of turbulent mixing

$$\frac{\partial \mu}{\partial S}\frac{(S-S_*(p_R))^2}{2}+\frac{c_p}{T_*}\frac{(T_R-T_*)^2}{2}$$



Summary

- Local theory of available potential energy naturally separates the potential energy into ‘work’ and ‘heat’ components
- Both ‘work’ and ‘heat’ components can be interpreted in terms of the energy involved in the various well defined thermodynamic transformations leading to the oceanic stratification
- Each positive definite component can itself be further partitioned into other positive definite components owing to additivity properties of thermodynamic potentials.

Summary cont’d

- APE dissipated is an irreversible, not reversible, conversion of APE into dead internal energy.
- Energetically consistent ocean models need not only correctly represent interactions between the ‘work’ reservoirs, they also need to correctly represent the related internal conversions between the different ‘heat’ reservoirs (Exergy, gravitational potential energy, and dead internal energy)

Postdoc position in Reading

To work on simple models of ocean heat uptake

with me, David Ferreira, Till Kuhlbrodt and Jonathan Gregory

Deadline: 23 May 2018

Interview date: 13 June 2018